A TABLE FOR THE CALCULATION OF WORKING PROBITS AND WEIGHTS IN PROBIT ANALYSIS

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The estimation of the parameters of a distribution of individual tolerances, from data relating to numbers of subjects manifesting a characteristic quantal response at different levels of a stimulus, is a problem frequently encountered in the application of statistical science to dose-mortality studies, biological assay, detonation of explosives, and other problems. A typical situation is that of exposing batches of insects to various doses of an insecticide, recording the proportion killed at each level of dose and then requiring to estimate the mean tolerance (or median lethal dose) of individual insects and the variance of the tolerance distribution. Gaddum (1933) and Bliss (1935*a*, *b*; 1938) have been instrumental in developing a method, that of the *probit transformation*, which greatly simplifies the calculations necessary to the estimation. The exact statistical analysis appropriate to the transformation was first shown by Fisher (1935), and the theory and uses of the method have been discussed fully in many subsequent publications (Finney, 1947*a*,*b*).

Tables required in the practice of the method, in sufficient detail for most purposes, have been given by various writers (Fisher & Yates, 1943; Finney, 1947*a*). Occasionally, however, the statistician needs values of the various functions at finer intervals of the argument, and for his benefit the following Table has been prepared. A brief account of the tabulated functions will suffice for all who are familiar with the probit method; those who require fuller information on the theory and analysis should consult the list of References.

Given a proportion P, and its complement $Q = 1 - P$, the probit of P is, to all intents and purposes, the deviate from the mean which divides the normal curve of unit variance in the ratio $P:Q$. In the formal definition, however, 5 is added to the deviate in order to avoid the necessity of computing with negative numbers. The advantage of this modification may be questioned, but it is now well established and will be adopted here. The probit, Y, of the proportion P is thus defined by

$$P = \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{Y-5} e^{-\frac{1}{2}u^2} du.$$

The standard method of analysis makes use of the *maximum and minimum working probits,*

$$Y_{max.} = Y + \frac{Q}{Z}$$

and

$$Y_{min.} = Y - \frac{P}{Z},$$

and also of the *range,* $1/Z$,

where

$$Z = \frac{1}{\sqrt{(2\pi)}} e^{-\frac{1}{2}(Y-5)^2}.$$

If n subjects receive the same stimulus, and r of them show the characteristic response, the empirical value for the proportion responding is

$$p = r/n;$$

the complement of this is denoted by $q = 1 - p$. The probits of a set of values of p should be approximately linearly related to x, the measure of the stimulus, and a line fitted by eye may be used to give a corresponding set of *expected probits*, Y. The *working probit* corresponding to each proportion is next calculated, from either

$$y = Y + Q/Z - q/Z,$$

or $$y = Y - P/Z + p/Z,$$

using tabulated values of the maximum or the minimum working probit (whichever is the more convenient) and the range. An improved set of expected probits is then derived from the weighted linear regression equation of working probits on x, each y being assigned a weight, nw, where the *weighting coefficient*, w, is defined as

$$w = Z^2/PQ.$$

The process may be repeated with the new set of Y values. The iteration converges to give a linear regression equation which is an estimate of

$$Y = 5 + (x - \mu)/\sigma,$$

where μ is the mean and σ the standard deviation of the tolerance distribution. The method depends upon an assumption that the stimulus is measured on a scale for which individual tolerances are normally distributed: often the logarithm of 'dose' rather than dose itself is taken as x, in order to satisfy this condition more closely.

The Table which follows gives $Y_{\max}$ for $Y = 3.58(0.01)\ 9.00$, $Y_{\min}$ for $Y = 1.00(0.01)$ 6.42, $1/Z$ and w for $Y = 1.00(0.01)\ 9.00$, all to four places of decimals. Below $Y = 3.58$, $Y_{\max}$ exceeds 10.00, and above $Y = 6.52$, $Y_{\min}$ is negative; it is then almost always more convenient to calculate working probits from the other function, but the function not tabulated can easily be obtained from the relationship

$$. \; Y_{\max} - Y_{\min} = 1/Z.$$

Between 3.58 and 6.42 both functions are tabulated. In order to save space the Table is arranged in parallel forward- and backward-reading columns; for the arguments Y and $(10 - Y)$ values of $1/Z$ and w are the same, and simple relations exist between $Y_{\max}$ and $Y_{\min}$. All entries have been calculated to six or more places of decimals and rounded to four, except that w between $Y = 2.7$ and $Y = 7.3$ was obtained by collating two existing tables and checking discrepancies.

The values of P and Z, from which the present Table has been calculated, were taken from *Tables of the Probability Function*, Vol. II (1942), published by the Federal Works Project Administration for the City of New York. Values of Q/Z have been taken from, or checked against, W. F. Sheppard's table, published as *The Probability Integral* (1939), Vol. VII of the *British Association Mathematical Tables*.

Example

In a batch of 281 insects receiving the same dose of insecticide, 119 are killed. The provisional probit regression line gives an expected probit of 4.61 for this dose; find the working probit and the weight to be attached to the observation.

Expected probit Y	Maximum working probit $Y+Q/Z$	Minimum working probit $Y-P/Z$	Range $1/Z$	Weighting coefficient Z^2/PQ			Y Expected probit
5·00	6·2533	3·7467	2·5066	0·6366	6·2533	3·7467	5·00
·01	·2534	·7466	·5068	·6366	·2534	·7466	4·99
·02	·2536	·7465	·5071	·6365	·2535	·7464	·98
·03	·2539	·7461	·5078	·6364	·2539	·7461	·97
·04	·2543	·7457	·5086	·6362	·2543	·7457	·96
5·05	6·2548	3·7450	2·5098	0·6360	6·2550	3·7452	4·95
·06	·2555	·7444	·5111	·6358	·2556	·7445	·94
·07	·2563	·7435	·5128	·6355	·2565	·7437	·93
·08	·2572	·7425	·5147	·6351	·2575	·7428	·92
·09	·2582	·7414	·5168	·6347	·2586	·7418	·91
5·10	6·2593	3·7401	2·5192	0·6343	6·2599	3·7407	4·90
·11	·2605	·7387	·5218	·6338	·2613	·7395	·89
·12	·2618	·7371	·5247	·6333	·2629	·7382	·88
·13	·2632	·7353	·5279	·6327	·2647	·7368	·87
·14	·2647	·7334	·5313	·6321	·2666	·7353	·86
5·15	6·2664	3·7314	2·5350	0·6314	6·2686	3·7336	4·85
·16	·2681	·7292	·5389	·6307	·2708	·7319	·84
·17	·2699	·7268	·5431	·6300	·2732	·7301	·83
·18	·2718	·7242	·5476	·6292	·2758	·7282	·82
·19	·2738	·7215	·5523	·6283	·2785	·7262	·81
5·20	6·2759	3·7186	2·5573	0·6274	6·2814	3·7241	4·80
·21	·2781	·7156	·5625	·6265	·2844	·7219	·79
·22	·2804	·7124	·5680	·6255	·2876	·7196	·78
·23	·2828	·7090	·5738	·6245	·2910	·7172	·77
·24	·2853	·7054	·5799	·6234	·2946	·7147	·76
5·25	6·2878	3·7016	2·5862	0·6223	6·2984	3·7122	4·75
·26	·2905	·6977	·5928	·6211	·3023	·7095	·74
·27	·2932	·6935	·5997	·6199	·3065	·7068	·73
·28	·2960	·6892	·6068	·6187	·3108	·7040	·72
·29	·2989	·6846	·6143	·6174	·3154	·7011	·71
5·30	6·3018	3·6798	2·6220	0·6161	6·3202	3·6982	4·70
·31	·3049	·6749	·6300	·6147	·3251	·6951	·69
·32	·3080	·6697	·6383	·6133	·3303	·6920	·68
·33	·3112	·6643	·6469	·6119	·3357	·6888	·67
·34	·3145	·6587	·6558	·6104	·3413	·6855	·66
5·35	6·3178	3·6528	2·6650	0·6088	6·3472	3·6822	4·65
·36	·3213	·6469	·6744	·6072	·3531	·6787	·64
·37	·3248	·6406	·6842	·6056	·3594	·6752	·63
·38	·3283	·6340	·6943	·6040	·3660	·6717	·62
·39	·3320	·6273	·7047	·6023	·3727	·6680	·61
5·40	6·3357	3·6203	2·7154	0·6005	6·3797	3·6643	4·60
·41	·3394	·6130	·7264	·5987	·3870	·6606	·59
·42	·3433	·6055	·7378	·5969	·3945	·6567	·58
·43	·3472	·5978	·7494	·5951	·4022	·6528	·57
·44	·3512	·5898	·7614	·5932	·4102	·6488	·56
5·45	6·3552	3·5815	2·7737	0·5912	6·4185	3·6448	4·55
·46	·3593	·5729	·7864	·5893	·4271	·6407	·54
·47	·3635	·5641	·7994	·5872	·4359	·6365	·53
·48	·3677	·5550	·8127	·5852	·4450	·6323	·52
·49	·3720	·5456	·8264	·5831	·4544	·6280	·51
5·50	6·3764	3·5360	2·8404	0·5810	6·4640	3·6236	4·50
			$1/Z$ Range	Z^2/PQ Weighting coefficient	$Y+Q/Z$ Maximum working probit	$Y-P/Z$ Minimum working probit	Y Expected probit

Expected probit Y	Maximum working probit $Y+Q/Z$	Minimum working probit $Y-P/Z$	Range $1/Z$	Weighting coefficient Z^2/PQ			
5·50	6·3764	3·5360	2·8404	0·5810	6·4640	3·6236	4·50
·51	·3808	·5260	·8548	·5788	·4740	·6192	·49
·52	·3852	·5157	·8695	·5766	·4843	·6148	·48
·53	·3898	·5052	·8846	·5744	·4948	·6102	·47
·54	·3944	·4943	·9001	·5722	·5057	·6056	·46
5·55	6·3990	3·4831	2·9159	0·5699	6·5169	3·6010	4·45
·56	·4037	·4715	·9322	·5675	·5285	·5963	·44
·57	·4085	·4597	·9488	·5652	·5403	·5915	·43
·58	·4133	·4475	·9658	·5628	·5525	·5867	·42
·59	·4181	·4349	·9832	·5603	·5651	·5819	·41
5·60	6·4230	3·4220	3·0010	0·5579	6·5780	3·5770	4·40
·61	·4280	·4088	·0192	·5554	·5912	·5720	·39
·62	·4330	·3952	·0378	·5529	·6048	·5670	·38
·63	·4381	·3812	·0569	·5503	·6188	·5619	·37
·64	·4432	·3669	·0763	·5477	·6331	·5568	·36
5·65	6·4484	3·3522	3·0962	0·5451	6·6478	3·5516	4·35
·66	·4536	·3370	·1166	·5425	·6630	·5464	·34
·67	·4588	·3214	·1374	·5398	·6786	·5412	·33
·68	·4641	·3055	·1586	·5371	·6945	·5359	·32
·69	·4695	·2892	·1803	·5343	·7108	·5305	·31
5·70	6·4749	3·2724	3·2025	0·5316	6·7276	3·5251	4·30
·71	·4803	·2551	·2252	·5288	·7449	·5197	·29
·72	·4858	·2375	·2483	·5260	·7625	·5142	·28
·73	·4914	·2194	··2720	·5232	·7806	··5086	·27
·74	·4969	·2008	·2961	·5203	·7992	·5031	·26
5·75	6·5026	3·1819	3·3207	0·5174	6·8181	3·4974	4·25
·76	·5082	·1623	·3459	·5145	·8377	·4918	·24
·77	·5139	·1423	·3716	·5116	·8577	·4861	·23
·78	·5197	·1219	·3978	·5086	·8781	·4803	·22
·79	·5255	·1009	·4246	·5056	·8991	·4745	·21
5·80	6·5313	3·0794	3·4519	0·5026	6·9206	3·4687	4·20
·81	·5372	·0574	·4798	·4996	·9426	·4628	·19
·82	·5431	·0348	·5083	·4965	·9652	·4569	·18
·83	·5490	·0116	·5374	·4935	·9884	·4510	·17
·84	·5550	2·9880	·5670	·4904	7·0120	·4450	·16
5·85	6·5611	2·9638	3·5973	0·4873	7·0362	3·4389	4·15
·86	·5671	·9389	·6282	·4841	·0611	·4329	·14
·87	·5732	·9135	·6597	·4810	·0865	·4268	·13
·88	·5794	·8875	·6919	·4778	·1125	·4206	·12
·89	·5855	·8608	·7247	·4746	·1392	·4145	·11
5·90	6·5917	2·8335	3·7582	0·4714	7·1665	3·4083	4·10
·91	·5980	·8056	·7924	·4682	·1944	·4020	·09
·92	·6043	·7771	·8272	·4650	·2229	·3957	·08
·93	·6106	·7478	·8628	·4617	·2522	·3894	·07
·94	·6169	·7178	·8991	·4585	·2822	·3831	·06
5·95	6·6233	2·6872	3·9361	0·4552	7·3128	3·3767	4·05
·96	·6297	·6558	·9739	·4519	·3442	·3703	·04
·97	·6362	·6238	4·0124	·4486	·3762	·3638	·03
·98	·6426	·5909	·0517	·4453	·4091	·3574	·02
·99	·6491	·5573	·0918	·4420	·4427	·3509	·01
6·00	6·6557	2·5230	4·1327	0·4386	7·4770	3·3443	4·00
			$1/Z$ Range	Z^2/PQ Weighting coefficient	$Y+Q/Z$ Maximum working probit	$Y-P/Z$ Minimum working probit	Y Expected probit

Expected probit Y	Maximum working probit $Y+Q/Z$	Minimum working probit $Y-P/Z$	Range $1/Z$	Weighting coefficient Z^2/PQ	Maximum working probit $Y+Q/Z$	Minimum working probit $Y-P/Z$	Expected probit Y
6·00	6·6557	2·5230	4·1327	0·4386	7·4770	3·3443	4·00
·01	·6623	·4878	·1745	·4353	·5122	·3377	3·99
·02	·6689	·4518	·2171	·4319	·5482	·3311	·98
·03	·6755	·4150	·2605	·4285	·5850	·3245	·97
·04	·6822	·3774	·3048	·4252	·6226	·3178	·96
6·05	6·6888	2·3387	4·3501	0·4218	7·6613	3·3112	3·95
·06	·6956	·2994	·3962	·4184	·7006	·3044	·94
·07	·7023	·2590	·4433	·4150	·7410	·2977	·93
·08	·7091	·2178	·4913	·4116	·7822	·2909	·92
·09	·7159	·1756	·5403	·4082	·8244	·2841	·91
6·10	6·7227	2·1324	4·5903	0·4047	7·8676	3·2773	3·90
·11	·7296	·0883	·6413	·4013	·9117	·2704	·89
·12	·7365	·0432	·6933	·3979	·9568	·2635	·88
·13	·7434	1·9970	·7464	·3944	8·0030	·2566	·87
·14	·7504	·9498	·8006	·3910	·0502	·2496	·86
6·15	6·7573	1·9014	4·8559	0·3876	8·0986	3·2427	3·85
·16	·7643	·8520	·9123	·3841	·1480	·2357	·84
·17	·7714	·8016	·9698	·3807	·1984	·2286	·83
·18	·7784	·7498	5·0286	·3772	·2502	·2216	·82
·19	·7855	·6970	·0885	·3738	·3030	·2145	·81
6·20	6·7926	1·6429	5·1497	0·3703	8·3571	3·2074	3·80
·21	·7997	·5876	·2121	·3669	·4124	·2003	·79
·22	·8068	·5310	·2758	·3634	·4690	·1932	·78
·23	·8140	·4731	·3409	·3600	·5269	·1860	·77
·24	·8212	·4140	·4072	·3565	·5860	·1788	·76
6·25	6·8284	1·3534	5·4750	0·3531	8·6466	3·1716	3·75
·26	·8357	·2916	·5441	·3496	·7084	·1643	·74
·27	·8429	·2282	·6147	·3462	·7718	·1571	·73
·28	·8502	·1635	·6867	·3428	·8365	·1498	·72
·29	·8575	·0972	·7603	·3393	·9028	·1425	·71
6·30	6·8649	1·0295	5·8354	0·3359	8·9705	3·1351	3·70
·31	·8722	0·9602	·9120	·3325	9·0398	·1278	·69
·32	·8796	·8893	·9903	·3291	·1107	·1204	·68
·33	·8870	·8168	6·0702	·3256	·1832	·1130	·67
·34	·8944	·7426	·1518	·3222	·2574	·1056	·66
6·35	6·9019	0·6668	6·2351	0·3188	9·3332	3·0981	3·65
·36	·9093	·5892	·3201	·3155	·4108	·0907	·64
·37	·9168	·5098	·4070	·3121	·4902	·0832	·63
·38	·9243	·4286	·4957	·3087	·5714	·0757	·62
·39	·0318	·3455	·5863	·3053	·6545	·0682	·61
6·40	6·9394	0·2606	6·6788	0·3020	9·7394	3·0606	3·60
·41	·9469	·1736	·7733	·2986	·8264	·0531	·59
·42	·9545	·0847	·8698	·2953	·9153	·0455	·58
·43	·9621		·9684	·2920		·0379	·57
·44	·9697		7·0691	·2887		·0303	·56
6·45	6·9774		7·1720	0·2854		3·0226	3·55
·46	·9850		·2771	·2821		·0150	·54
·47	6·9927		·3845	·2788		·0073	·53
·48	7·0004		·4943	·2756		2·9996	·52
·49	·0081		·6064	·2723		·9919	·51
6·50	7·0158		7·7210	0·2691		2·9842	3·50

Expected probit Y	Maximum working probit Y+Q/Z	Range 1/Z	Weighting coefficient Z²/PQ		
6·50	7·0158	7·7210	0·2691	2·9842	3·50
·51	·0236	·8380	·2658	·9764	·49
·52	·0313	·9577	·2626	·9687	·48
·53	·0391	8·0800	·2594	·9609	·47
·54	·0469	·2050	·2563	·9531	·46
6·55	7·0547	8·3327	0·2531	2·9453	3·45
·56	·0625	·4633	·2500	·9375	·44
·57	·0704	·5968	·2468	·9296	·43
·58	·0783	·7333	·2437	·9217	·42
·59	·0861	·8728	·2406	·9139	·41
6·60	7·0940	9·0154	0·2375	2·9060	3·40
·61	·1020	·1613	·2345	·8980	·39
·62	·1099	·3105	·2314	·8901	·38
·63	·1178	·4630	·2284	·8822	·37
·64	·1258	·6190	·2254	·8742	·36
6·65	7·1338	9·7785	0·2224	2·8662	3·35
·66	·1417	·9417	·2194	·8583	·34
·67	·1498	10·1086	·2165	·8502	·33
·68	·1578	10·2794	·2135	·8422	·32
·69	·1658	10·4540	·2106	·8342	·31
6·70	7·1739	10·6327	0·2077	2·8261	3·30
·71	·1819	10·8156	·2049	·8181	·29
·72	·1900	11·0027	·2020	·8100	·28
·73	·1981	11·1941	·1992	·8019	·27
·74	·2062	11·3900	·1964	·7938	·26
6·75	7·2143	11·5905	0·1936	2·7857	3·25
·76	·2224	11·7957	·1908	·7776	·24
·77	·2306	12·0058	·1881	·7694	·23
·78	·2387	12·2208	·1853	·7613	·22
·79	·2469	12·4409	·1826	·7531	·21
6·80	7·2551	12·6662	0·1799	2·7449	3·20
·81	·2633	12·8969	·1773	·7367	·19
·82	·2715	13·1331	·1746	·7285	·18
·83	·2797	13·3750	·1720	·7203	·17
·84	·2880	13·6227	·1694	·7120	·16
6·85	7·2962	13·8764	0·1669	2·7038	3·15
·86	·3045	14·1362	·1643	·6955	·14
·87	·3128	14·4023	·1618	·6872	·13
·88	·3210	14·6749	·1593	·6790	·12
·89	·3293	14·9541	·1568	·6707	·11
6·90	7·3376	15·2402	0·1544	2·6624	3·10
·91	·3460	15·5333	·1519	·6540	·09
·92	·3543	15·8337	·1495	·6457	·08
·93	·3626	16·1414	·1471	·6374	·07
·94	·3710	16·4568	·1448	·6290	·06
6·95	7·3794	16·7800	0·1424	2·6206	3·05
·96	·3877	17·1113	·1401	·6123	·04
·97	·3961	17·4509	·1378	·6039	·03
·98	·4045	17·7989	·1356	·5955	·02
·99	·4129	18·1558	·1333	·5871	·01
7·00	7·4214	18·5216	0·1311	2·5786	3·00
		1/Z Range	Z²/PQ Weighting coefficient	Y−P/Z Minimum working probit	Y Expected probit

Expected probit Y	Maximum working probit $Y + Q/Z$	Range $1/Z$	Weighting coefficient Z^2/PQ		
7·00	7·4214	18·5216	0·1311	2·5786	3·00
·01	·4298	18·8967	·1289	·5702	2·99
·02	·4382	19·2814	·1268	·5618	·98
·03	·4467	19·6758	·1246	·5533	·97
·04	·4552	20·0803	·1225	·5448	·96
7·05	7·4636	20·4952	0·1204	2·5364	2·95
·06	·4721	20·9207	·1183	·5279	·94
·07	·4806	21·3572	·1163	·5194	·93
·08	·4891	21·8050	·1142	·5109	·92
·09	·4976	22·2644	·1122	·5024	·91
7·10	7·5062	22·7357	0·1103	2·4938	2·90
·11	·5147	23·2194	·1083	·4853	·89
·12	·5232	23·7157	·1064	·4768	·88
·13	·5318	24·2251	·1045	·4682	·87
·14	·5404	24·7478	·1026	·4596	·86
7·15	7·5489	25·2844	0·1007	2·4511	2·85
·16	·5575	25·8352	·0989	·4425	·84
·17	·5661	26·4006	·0971	·4339	·83
·18	·5747	26·9812	·0953	·4253	·82
·19	·5833	27·5772	·0935	·4167	·81
7·20	7·5919	28·1892	0·0918	2·4081	2·80
·21	·6006	28·8177	·0901	·3994	·79
·22	·6092	29·4631	·0884	·3908	·78
·23	·6178	30·1260	·0867	·3822	·77
·24	·6265	30·8069	·0851	·3735	·76
7·25	7·6351	31·5063	0·0834	2·3649	2·75
·26	·6438	32·2249	·0818	·3562	·74
·27	·6525	32·9631	·0802	·3475	·73
·28	·6612	33·7216	·0787	·3388	·72
·29	·6699	34·5010	·0771	·3301	·71
7·30	7·6786	35·3020	0·0756	2·3214	2·70
·31	·6873	36·1251	·0741	·3127	·69
·32	·6960	36·9712	·0727	·3040	·68
·33	·7047	37·8408	·0712	·2953	·67
·34	·7135	38·7348	·0698	·2865	·66
7·35	7·7222	39·6539	0·0684	2·2778	2·65
·36	·7310	40·5988	·0671	·2690	·64
·37	·7397	41·5704	·0656	·2603	·63
·38	·7485	42·5695	·0643	·2515	·62
·39	·7573	43·5970	·0630	·2427	·61
7·40	7·7661	44·6538	0·0617	2·2339	2·60
·41	·7748	45·7407	·0604	·2252	·59
·42	·7836	46·8588	·0591	·2164	·58
·43	·7924	48·0090	·0579	·2076	·57
·44	·8013	49·1924	·0567	·1987	·56
7·45	7·8101	50·4099	0·0555	2·1899	2·55
·46	·8189	51·6628	·0543	·1811	·54
·47	·8277	52·9521	·0532	·1723	·53
·48	·8366	54·2791	·0520	·1634	·52
·49	·8454	55·6448	·0509	·1546	·51
7·50	7·8543	57·0506	0·0498	2·1457	2·50
		$1/Z$ Range	Z^2/PQ Weighting coefficient	$Y - P/Z$ Minimum working probit	Y Expected probit

Expected probit Y	Maximum working probit $Y+Q/Z$	Range $1/Z$	Weighting coefficient Z^2/PQ		
7·50	7·8543	57·0506	0·0498	2·1457	2·50
·51	·8631	58·4978	·0487	·1369	·49
·52	·8720	59·9876	·0476	·1280	·48
·53	·8809	61·5216	·0466	·1191	·47
·54	·8897	63·1011	·0456	·1103	·46
7·55	7·8986	64·7277	0·0446	2·1014	2·45
·56	·9075	66·4028	·0436	·0925	·44
·57	·9164	68·1280	·0426	·0836	·43
·58	·9253	69·9051	·0416	·0747	·42
·59	·9342	71·7357	·0407	·0658	·41
7·60	7·9432	73·6216	0·0398	2·0568	2·40
·61	·9521	75·5646	·0389	·0479	·39
·62	·9610	77·5667	·0380	·0390	·38
·63	·9700	79·6298	·0371	·0300	·37
·64	·9789	81·7559	·0362	·0211	·36
7·65	7·9879	83·9472	0·0354	2·0121	2·35
·66	·9968	86·2059	·0346	·0032	·34
·67	8·0058	88·5342	·0338	1·9942	·33
·68	·0147	90·9344	·0330	·9853	·32
·69	·0237	93·4091	·0322	·9763	·31
7·70	8·0327	95·9607	0·0314	1·9673	2·30
·71	·0417	98·5918	·0307	·9583	·29
·72	·0507	101·3053	·0300	·9493	·28
·73	·0597	104·1038	·0292	·9403	·27
·74	·0687	106·9903	·0285	·9313	·26
7·75	8·0777	109·9679	0·0278	1·9223	2·25
·76	·0867	113·0396	·0272	·9133	·24
·77	·0957	116·2088	·0265	·9043	·23
·78	·1047	119·4788	·0258	·8953	·22
·79	·1138	122·8530	·0252	·8862	·21
7·80	8·1228	126·3352	0·0246	1·8772	2·20
·81	·1318	129·9290	·0240	·8682	·19
·82	·1409	133·6385	·0234	·8591	·18
·83	·1499	137·4676	·0228	·8501	·17
·84	·1590	141·4206	·0222	·8410	·16
7·85	8·1681	145·5018	0·0217	1·8319	2·15
·86	·1771	149·7158	·0211	·8229	·14
·87	·1862	154·0671	·0206	·8138	·13
·88	·1953	158·5609	·0200	·8047	·12
·89	·2044	163·2020	·0195	·7956	·11
7·90	8·2134	167·9957	0·0190	1·7866	2·10
·91	·2225	172·9476	·0185	·7775	·09
·92	·2316	178·0632	·0181	·7684	·08
·93	·2407	183·3485	·0176	·7593	·07
·94	·2498	188·8095	·0171	·7502	·06
7·95	8·2590	194·4526	0·0167	1·7410	2·05
·96	·2681	200·2844	·0162	·7319	·04
·97	·2772	206·3118	·0158	·7228	·03
·98	·2863	212·5418	·0154	·7137	·02
·99	·2955	218·9818	·0150	·7045	·01
8·00	8·3046	225·6395	0·0146	1·6954	2·00
		$1/Z$ Range	Z^2/PQ Weighting coefficient	$Y-P/Z$ Minimum working probit	Y Expected probit

Expected probit Y	Maximum working probit $Y + Q/Z$	Range $1/Z$	Weighting coefficient Z^2/PQ		
8·00	8·3046	225·6395	0·0146	1·6954	2·00
·01	·3137	232·5229	·0142	·6863	1·99
·02	·3229	239·6402	·0138	·6771	·98
·03	·3320	247·0000	·0134	·6680	·97
·04	·3412	254·6114	·0131	·6588	·96
8·05	8·3503	262·4836	0·0127	·1·6497	1·95
·06	·3595	270·6262	·0124	·6405	·94
·07	·3687	279·0493	·0120	·6313	·93
·08	·3778	287·7634	·0117	·6222	·92
·09	·3870	296·7792	·0114	·6130	·91
8·10	8·3962	306·1082	0·0110	1·6038	1·90
·11	·4054	315·7619	·0107	·5946	·89
·12	·4146	325·7527	·0104	·5854	·88
·13	·4238	336·0932	·0101	·5762	·87
·14	·4330	346·7966	·0099	·5670	·86
8·15	8·4422	357·8732	0·0096	1·5578	1·85
·16	·4514	369·3477	·0093	·5486	·84
·17	·4606	381·2245	·0090	·5394	·83
·18	·4698	393·5226	·0088	·5302	·82
·19	·4790	406·2580	·0085	·5210	·81
8·20	8·4882	419·4476	0·0083	1·5118	1·80
·21	·4974	433·1086	·0080	·5026	·79
·22	·5067	447·2593	·0078	·4933	·78
·23	·5159	461·9185	·0076	·4841	·77
·24	·5251	477·1059	·0074	·4749	·76
8·25	8·5344	492·8419	0·0071	1·4656	1·75
·26	·5436	509·1479	·0069	·4564	·74
·27	·5529	526·0459	·0067	·4471	·73
·28	·5621	543·5592	·0065	·4379	·72
·29	·5714	561·7116	·0063	·4286	·71
8·30	8·5806	580·5283	0·0061	1·4194	1·70
·31	·5899	600·0353	·0060	·4101	·69
·32	·5992	620·2599	·0058	·4008	·68
·33	·6084	641·2302	·0056	·3916	·67
·34	·6177	662·9758	·0054	·3823	·66
8·35	8·6270	685·5274	0·0053	1·3730	1·65
·36	·6363	708·9171	·0051	·3637	·64
·37	·6456	733·1780	·0050	·3544	·63
·38	·6548	758·3451	·0048	·3452	·62
·39	·6641	784·4545	·0047	·3359	·61
8·40	8·6734	811·5439	0·0045	1·3266	1·60
·41	·6827	839·6528	·0044	·3173	·59
·42	·6920	868·8222	·0042	·3080	·58
·43	·7013	899·0948	·0041	·2987	·57
·44	·7106	930·5153	·0040	·2894	·56
8·45	8·7200	963·1301	0·0038	1·2800	1·55
·46	·7293	996·9878	·0037	·2707	·54
·47	·7386	1032·1389	·0036	·2614	·53
·48	·7479	1068·6362	·0035	·2521	·52
·49	·7572	1106·5347	·0034	·2428	·51
8·50	8·7666	1145·8919	0·0033	1·2334	1·50
		$1/Z$ Range	Z^2/PQ Weighting coefficient	$Y - P/Z$ Minimum working probit	Y Expected probit

Expected probit Y	Maximum working probit $Y + Q/Z$	Range $1/Z$	Weighting coefficient Z^2/PQ		
8·50	8·7666	1145·8919	0·0033	1·2334	1·50
·51	·7759	1186·7675	·0032	·2241	·49
·52	·7852	1229·2242	·0031	·2148	·48
·53	·7946	1273·3271	·0030	·2054	·47
·54	·8039	1319·1443	·0029	·1961	·46
8·55	8·8133	1366·7467	0·0028	1·1867	1·45
·56	·8226	1416·2085	·0027	·1774	·44
·57	·8320	1467·6071	·0026	·1680	·43
·58	·8413	1521·0232	·0025	·1587	·42
·59	·8507	1576·5411	·0024	·1493	·41
8·60	8·8600	1634·2488	0·0024	1·1400	1·40
·61	·8694	1694·2383	·0023	·1306	·39
·62	·8788	1756·6055	·0022	·1212	·38
·63	·8881	1821·4507	·0021	·1119	·37
·64	·8975	1888·8785	·0021	·1025	·36
8·65	8·9069	1958·9983	0·0020	1·0931	1·35
·66	·9162	2031·9243	·0019	·0838	·34
·67	·9256	2107·7758	·0019	·0744	·33
·68	·9350	2186·6775	·0018	·0650	·32
·69	·9444	2268·7596	·0017	·0556	·31
8·70	8·9538	2354·1583	0·0017	1·0462	1·30
·71	·9632	2443·0158	·0016	·0368	·29
·72	·9726	2535·4807	·0016	·0274	·28
·73	·9820	2631·7085	·0015	·0180	·27
·74	·9914	2731·8615	·0015	·0086	·26
8·75	9·0008	2836·1096	0·0014	0·9992	1·25
·76	·0102	2944·6302	·0014	·9898	·24
·77	·0196	3057·6091	·0013	·9804	·23
·78	·0290	3175·2401	·0013	·9710	·22
·79	·0384	3297·7264	·0012	·9616	·21
8·80	9·0478	3425·2801	0·0012	0·9522	1·20
·81	·0572	3558·1233	·0011	·9428	·19
·82	·0667	3696·4883	·0011	·9333	·18
·83	·0761	3840·6179	·0011	·9239	·17
·84	·0855	3990·7662	·0010	·9145	·16
8·85	9·0949	4147·1994	0·0010	0·9051	1·15
·86	·1044	4310·1955	·0010	·8956	·14
·87	·1138	4480·0457	·0009	·8862	·13
·88	·1232	4657·0549	·0009	·8768	·12
·89	·1327	4841·5419	·0009	·8673	·11
8·90	9·1421	5033·8407	0·0008	0·8579	1·10
·91	·1516	5234·3007	·0008	·8484	·09
·92	·1610	5443·2878	·0008	·8390	·08
·93	·1704	5661·1851	·0007	·8296	·07
·94	·1799	5888·3938	·0007	·8201	·06
8·95	9·1894	6125·3338	0·0007	0·8106	1·05
·96	·1988	6372·4452	·0007	·8012	·04
·97	·2083	6630·1886	·0006	·7917	·03
·98	·2177	6899·0468	·0006	·7823	·02
·99	·2272	7179·5252	·0006	·7728	·01
9·00	9·2367	7472·1536	0·0006	0·7633	1·00
		$1/Z$ Range	Z^2/PQ Weighting coefficient	$Y - P/Z$ Minimum working probit	Y Expected probit

The proportion killed is $\quad p = 119/281 = 0\cdot4235$.

For $Y = 4\cdot61$, the Table shows the minimum working probit and range as

$$Y_{min.} = 3\cdot6680,$$

and $\qquad\qquad 1/Z = 2\cdot7047.$

Hence the working probit is $\quad Y = 3\cdot6680 + 2\cdot7047p$

$$= 4\cdot8134.$$

Alternatively, if survivors instead of deaths have been recorded, calculation may proceed from

$$Y_{max.} = 6\cdot3727,$$

giving $\qquad\qquad y = 6\cdot3727 - 2\cdot7047 \times 0\cdot5765$

$$= 4\cdot8134.$$

The Table shows $\qquad w = 0\cdot6023,$

so that the weight for the observation is

$$nw = 281 \times 0\cdot6023$$

$$= 169\cdot2.$$

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