

## WMS5 Exercises

2.2 , 2.5, 2.6 , p23

2.8 , 2.9 , 2.10 , 2.11 , p29

2.14 , 2.15 , 2.16 , p34

2.21 , 2.23 , 2.25 , 2.32 , 2.39 , p44

2.48 , p48 ; 2.49, 2.50 , p49

2.58 , p52, 2.61 , p52; 2.62 , 2.65 , 2.67 , p53

2.70 , 2.71 , 2.72 , 2.74 , 2.76 , p60; 2.81 , p61

2.83 , p63

*Note: this is a non-standard use of the word "accuracy". In describing the performance of a medical diagnostic test, we speak of "the probability that the test is positive given that a person has the disease" as the "Sensitivity" of the test; we speak of "the probability that the test is negative given that a person does not have the disease" as the "Specificity" of the test. There is nothing that says the sensitivity must equal the specificity.*

2.86 , p63; 2.89 , p64

2.110 , 2.111 , 2.113 , p72; 2.117, 2.120 , p73

## Exercises from M&M3

4.1 Hold a penny upright on its edge under your forefinger on a hard surface, then snap it with your other forefinger so that it spins for some time before falling. Based on 50 spins, what is the probability of heads?

4.2 You may feel that it is obvious that the probability of a head in tossing a coin is about  $1/2$  because the coin has two faces. Such opinions are not always correct. The previous exercise asked you to spin a penny rather than toss it—that changes the probability of a head. Now try another variation. Stand a penny on edge on a flat surface. Pound the surface with your hand so that the penny falls over. What is the probability that it falls with heads upward? Make at least 50 trials to estimate the probability of a head.

4.4 Toss a thumbtack on a hard surface 100 times. How many times did it land with the point up? What is the approximate probability of landing point up?

4.5 Obtain 10 identical thumbtacks and toss all 10 at once. Do this 50 times and record the number that land point up on each trial.

- (a) what is the approx. probability that at least one lands point up?  
(b) what is the approx. probability that more than one lands point up?

### Homegrown Exercises

Q1 See on the class web site the requirements for the various prizes for the loto-québec lottery game **La Quotidienne 3**.

A Assume that a player has chosen the "EXACT ORDER" type of wager. What is the probability of winning the prize if the player has chosen

- the digits 3 7 1?
- the digits 0 0 0?
- the digits 7 7 7?

B Assume that a player has chosen the "ANY ORDER" type of wager. What is the probability of winning the prize if the player has chosen

- 3 identical digits (ex.: 1 1 1)?
- 2 identical digits (ex.: 1 1 2)?
- 3 different digits (ex.: 1 2 3)?

Q2 See on the class web site the requirements for the various prizes for the loto-québec lottery game **La Quotidienne 4**.

Assume that a player has chosen the "ANY ORDER" type of wager. What is the probability of winning the prize if the player has chosen

- 4 identical digits (ex.: 1 1 1 1)?
- 3 identical digits (ex.: 1 1 1 2)?
- 2 pairs (ex.: 1 1 2 2)
- 2 identical digits (ex.: 1 1 2 3)?
- 4 different digits (ex.: 1 2 3 4)?

3 See on the class web site the procedure for playing the loto-québec lottery game **COMBINATIONS Super 7**. The Instructions at the left hand side of the front of the form state that "A combination of 6, 8 or 9 numbers corresponds to 41, 8 or 36 selections of 7 numbers each"

Explain how these numbers 41, 8 and 36 are arrived at. (See also the sentence "If 6 numbers are selected.." and the sentence "If 8 or 9 numbers are selected,..." in the paragraph that begins with "To participate, ..." on the back of the form.)

**Exercises from MRT2**

13D6 In 1986, about 51% of the U.S. populations was female. Also, about 12% of the population was age 65 and over. true or false, and explain: The percentage of the population consisting of women age 65 and over is

$$51\% \text{ of } 12\% = 0.51 \times 12\% \quad 6\%.$$

13.7.4 A deck of cards is thoroughly shuffled. You have to deal one card at a time, until an ace turns up.

- You have gone through 10 cards, and still have not seen an ace. What is the chance of getting an ace on the 11th card?
- What is the probability that the first ace appears  
(a) on the 5th card? (b) at the k-th card (c) at the k-th card or sooner?

13.7.8 A coin is tossed 6 times. Two possible sequences are

- (i) H T T H T H      (ii) H H H H H H

(the coin must land H or T in the order given: H = heads, T = tails.) Which of the following is correct? Explain.

- (a) Sequence (i) is more likely.
- (b) Sequence (ii) is more likely.
- (c) Both sequences are equally likely.

**Exercises from MRT2**

1.6.1 To appreciate the importance of experimental verification as well as clear reasoning (including labeling each item separately), consider the experiment of tossing two coins. It is said that the mathematician D'Alembert reasoned that there are three possible ways the coins can fall: (a) both heads, (b) one head and one tail, and (c) both tails. He thought these three outcomes were equally likely, and expected that each of them would occur about 1/3 of the time in actual experiments.

Toss 2 coins 50 times, keeping a careful tally of the possible outcomes—two heads, one head and one tail, two tails. Compute the proportion of occurrence of each of the three outcomes,. Do your results appear to support the reasoning of D'Alembert that the three outcomes are equally likely?

1.7.1 A coin is tossed and then a die is thrown. List the sample space for this experiment. Illustrate with a tree graph.

1.7.7 Two dice, one black and one red, are tossed and the numbers of dots on their upper faces are noted. List the sample space for this experiment. [Note: A tabular arrangement is convenient].

How many sample points correspond to

- a total of more than 10 dots?
- a total of less than 5 dots?
- an even total?
- the red die showing less than 3 and the black die showing more than 5?
- the red die showing less than 3 and the black die showing 3?

1.7.11 In the ancient Indian game of Tong, two players simultaneously show their right hands to each other, exhibiting either one or two or three extended fingers. If each player is equally likely to extend one, two or three fingers, what is the probability that the total number of fingers extended is • even? • odd? • greater than 4? • less than 2?

p32.6 You ask a friend to "think of a number". Describe the sample space for this experiment.

p32.12 A plant breeder crosses two parent strains, each possessing a gene pair of type  $aA$ . Each parent contributes one-half this gene pair (either  $a$  or  $A$ ) to the offspring, where the two halves are combined. Describe the sample space of the offspring.

2.1.3 If the call letters of a broadcasting station must begin with the letter C, how many different stations can be designated by using only 3 letters, with repetitions of a letter allowed? How many by using 4 letters, without repetitions?

4.2.5 The true odds in favour of three events that are mutually exclusive, and whose union is the sample space, are in the ratio 3 to 2 to 1. Find the probabilities of the three events.

4.2.7 The probabilities of the mutually exclusive events A and B are related as  $P(B) = [P(A)]^2$ , and  $A \cup B = S$ , the sample space. Find  $P(A)$  (approx.) [read " $A \cup B$ " as " $A$  union  $B$ "]