

## Preamble

- Should not be in same chapter with confounding...
- a very different topic !! (can have both, but ... see diagram)

## Definitions ...

Interaction (statistical)

- "Non-additivity" of "effects" in regression
- need for product term in regression analysis (osm)
  - scale dependent

(Effect) Modification (epidemiological)

- Inconstancy of a parameter of a relation over other subject characteristic (osm)
- Different slopes for different folks (jh)

"Modifier (of a relation)

- A characteristic (of individuals) on which a parameter of a relation depends (osm)

## Examples...

- Equation for Ideal Weight as function of Height

- *modification by Gender*

- Average Earnings as function of Education / Age

- *modification by Gender*

- Decline in Bone Density with Age

- *Different in 19th and 20th Centuries*

- ?Can hit further with aluminum than wood baseball bat?

- *Difference depends on where on bat one hits ball*

- Changes over time in injury rates

- *Different in intervention and reference areas?*

## Translating these into regression equations ...

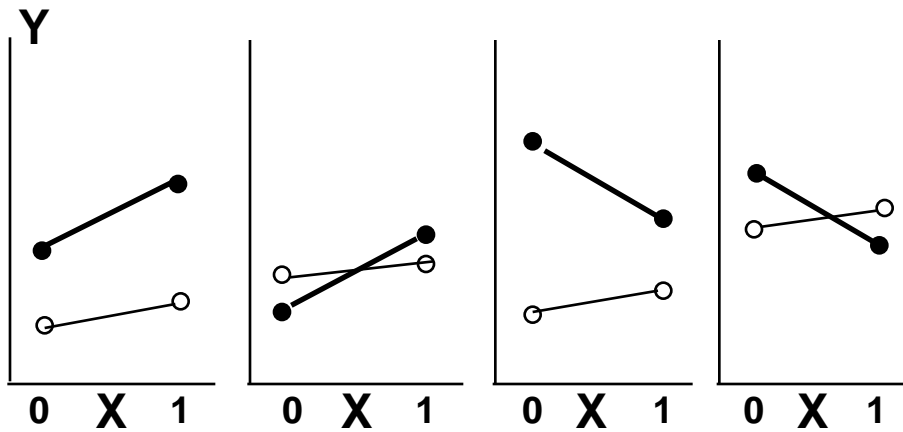
- relation between Y and X

- "modifier" variable M

$$E[Y | X, M] = B_0 + B_1.X + B_2.M + B_3.(M.X)$$

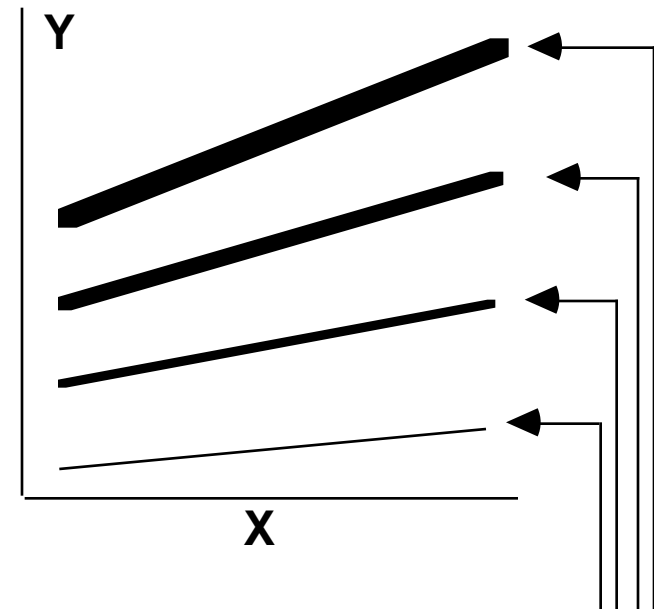
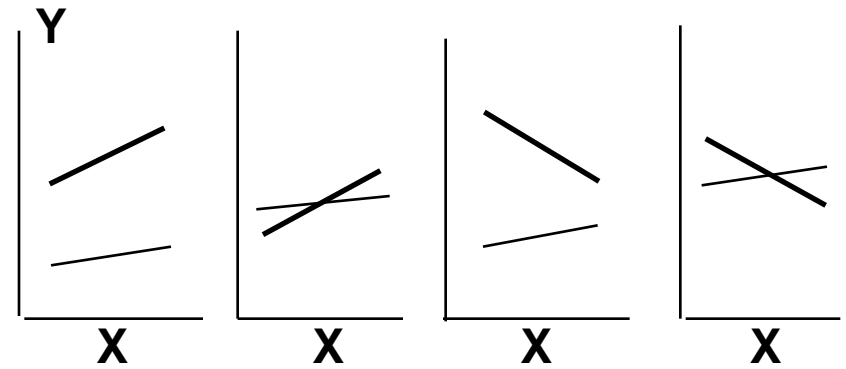
- Special cases..

### X binary, M Binary



————— **Modifier = 1**  
 ————— **Modifier = 0**

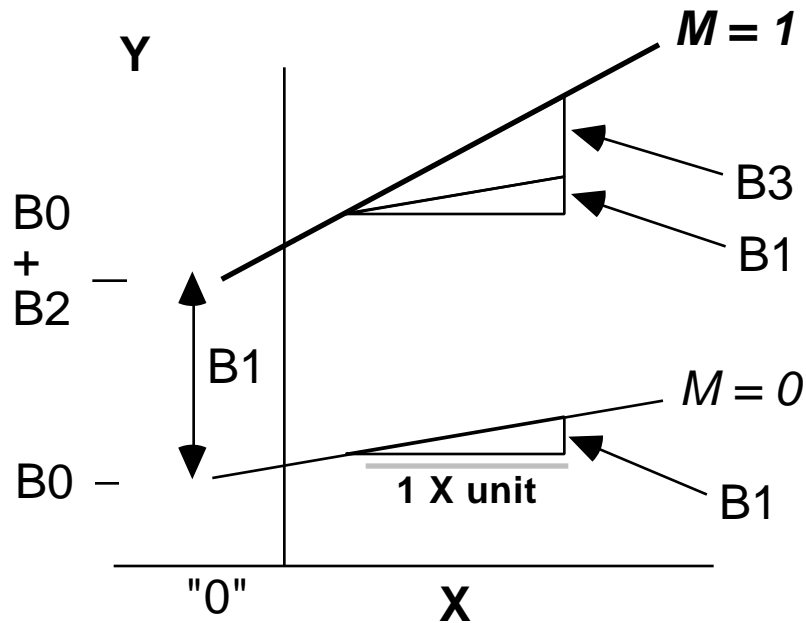
### X continuous, M binary



**Quantitative levels of Modifier M**

## Meaning of the coefficients

X continuous, M Binary



- helpful ways of rewriting the equation

$$E[Y | X, M] = B_0 + B_2.M + (B_1 + B_3.M).X$$

## Special issues

- mathematical symmetry of equation

$$E[Y | X_1, X_2] = B_0 + B_1.X_1 + B_2.X_2 + B_3.(X_1.X_2)$$

$$= B_0 + B_2.X_2 + (B_1 + B_3.X_2).X_1$$

*X2 modifies the Y<->X1 relation*

$$= B_0 + B_1.X_1 + (B_2 + B_3.X_1).X_2$$

*X1 modifies the Y<->X2 relation*

- to a regression program, X1.X2 product terms are

just like any other terms.. but

they tend to be correlated (collinear) with the

components from which they are made, so...

\*\*\* user should "center" the components before \*\*\*

\*\*\* making (or having computer make) products \*\*\*

*(will see example in injury prevention study)*

## Translating equations back into lines ...

- **If M is binary...**

start with the M=0 case

$$B_0 + B_1.X + B_2.M + B_3.(M.X)$$

$$= B_0 + B_1.X + B_2.0 + B_3.(0.X)$$

$$= B_0 + B_1.X$$

====> *straight line in X with intercept B<sub>0</sub> and slope B<sub>1</sub>*

"turn on" the M=1 toggle...

$$B_0 + B_1.X + B_2.M + B_3.(M.X)$$

$$= B_0 + B_1.X + B_2.1 + B_3.(1.X)$$

$$= B_0 + B_1.X + B_2 + B_3.X$$

collect terms that do not involve X & those that do..

$$(B_0 + B_2) + (B_1 + B_3).X$$

====> *straight line in X with intercept (B<sub>0</sub> + B<sub>2</sub>) and slope (B<sub>1</sub> + B<sub>3</sub>)*

- **If M is continuous...** as above with several M values