Simpson’s Paradox
An example in a New Zealand Survey of Jury Composition

Research Report # 1

(Revised)

Simpson's Paradox

An example in a New Zealand Survey of Jury Composition

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A Paradoxical Result

I came across an interesting problem when I was asked to review a draft report on a survey of jury composition in New Zealand.

In September 1993, the New Zealand Department of Justice surveyed the composition of juries and of the pool of jurors the juries were selected from. Of particular interest was the question of representation of Maori, the indigenous people of New Zealand. The survey covered all potential jurors who arrived at court during a period in September and October 1993. It produced some very interesting results, published in 1995 in *Trial By Peers*. Looking at the results nationally, the report notes "9.5 percent of people living within the jury districts were Maori. This compares with 10.1 percent of Maori in the pool of potential jurors. It is tempting to conclude, therefore, that Maori were adequately represented in the jury pool".

This is a rather unexpected result, given the original impetus for the research - that Maori appeared to be under-represented on juries. Is all as it seems? I thought I'd delve a little.

The draft report gave the Maori proportion for each court district. I expected some variation, so I thought why not have a look to see which areas were above and which were below the proportion in the population. Fortunately, I had access to 1991 Census data. The results, shown in Table 1, gave me a big surprise.

Overall, Maori appeared to be slightly over-represented in the jury pool. But in every single local area, Maori were under-represented - often substantially. That is, the proportion of Maori in the population eligible for jury service was greater than the proportion in the jury pool. This is an example of Simpson's paradox.

Table 1 : The Paradox

Maori, overall, appear to be over-represented. Yet in every district they are under-represented.

Percentage Maori ethnic group

District	Eligible Population (aged 20-64)	Jury Pool	Shortfall
Whangarei	17.0	16.8	0.2
Auckland	9.2	9.0	0.2
Hamilton	13.5	11.5	2.0
Rotorua	27.0	23.4	3.6
Gisborne	32.2	29.5	2.7
Napier	15.5	12.4	3.1
New Plymouth	8.9	4.1	4.8
Palmerston North	8.9	4.3	4.6
Wellington	8.7	7.5	1.2
Nelson	3.9	1.7	2.2
Christchurch	4.5	3.3	1.2
Dunedin	3.3	2.4	0.9
Invercargill	8.4	4.8	3.6
All Districts	**9.5**	**10.1**	**-0.6**

Source: Unpublished 1991 Census tables, and Trial By Peers

What is Simpson's Paradox

Simpson's paradox occurs when combining data sets gives a misleading overall picture. The name originates from a paper by Simpson in 1951.

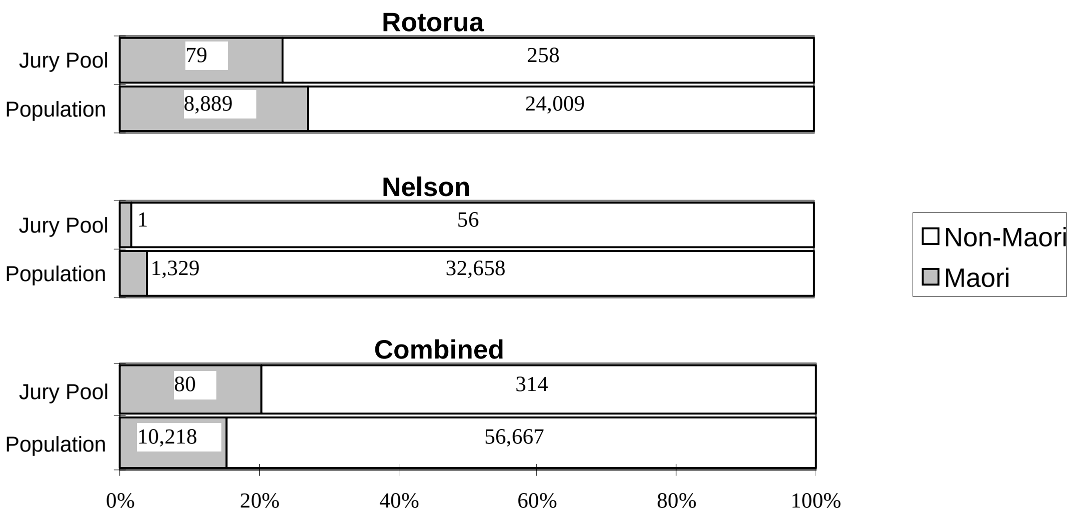
Let's have a look at what happens in this case, as it is a good case study for the more general problem. A genuine example in fact - virtually all the examples I've seen in the literature look distinctly artificial. Another real example, "proving" that smoking is beneficial, is given by Appleton *et al.* (1996).

How does it happen

To see what is happening consider just two cities. Rotorua and Nelson are about the same size overall - but have big differences in the proportion of Maori, as shown in Figure 1. For both the jury and population Rotorua has more than 20 percent Maori, while for Nelson both percentages are in the low single figures. But in each case, Maori are under-represented in the jury pool. However, when we combine the data for the two cities, and take the percentages again, Maori appear to be over-represented in the pool.

Figure 1

What happens with just two cities



In this case, the much larger size of the jury pool in Rotorua pulls up the proportion of Maori in the combined jury pool.

The paradox arises because we assume that combining the data will average the proportions. In this case we expect that combining will average the proportions of Maori in each of the jury pools, and likewise average the proportions of Maori in each of the populations. Combining the data does create an average - but a weighted not a simple average. The proportion in the combined data is the average of the proportions in each group contributing, weighted by the size of the contribution to the combined data. In the two city example, Rotorua contributes 32,898 to the combined population while Nelson contributes 33,987, so their weights are almost equal. However, Rotorua contributes 337 to the jury pool, while Nelson contributes only 57, or under 15 percent. A more formal explanation is given in Blyth (1972).

If each district contributed in the same proportions to the jury pool and the population, there could be no paradox. However, those districts with a high proportion of Maori tend to have relatively large jury pools, and this pulls the proportion of Maori in the jury pool for the combined districts above the proportion of Maori in the population.

How to present

We can see that Simpson's paradox is well named. The effect described is paradoxical when we approach the problem as we have so far. It is rather difficult to explain clearly.

How can the results be presented effectively to a general audience? It's better to steer clear of weighted averages and the like. And I didn't think presenting the apparently contradictory tables we've seen would help a lot either. Instead, let's consider the expected number of Maori in the jury pool if the selection was random for this effect. The final report used this approach, leading to the publication of Table 2 in *Trial By Peers*.

Table 2

Actual Number of Maori in Jury Pool
Compared with the Expected Number

	Actual	Expected
Whangarei	28	28
Auckland	74	76
Hamilton	23	27
Rotorua	79	91
Gisborne	23	25
Napier	15	19
New Plymouth	4	9
Palmerston North	7	14
Wellington	28	33
Nelson	1	2
Christchurch	11	15
Dunedin	4	6
Invercargill	3	5
Total	**300**	**350**

Source: *Trial By Peers*

Table 2 shows the actual number of Maori in the jury pool in the first column, compared with the expected number if Maori were represented according to their proportion in the local population. This approach has one big advantage: the numbers of jurors - actual and expected can be added to get an overall total that means something. The percentages used before hid the real situation of Maori under-representation. These totals show the under-representation at the national level as well as at every local level bar one.

The final publication picked up on this approach, and gave these conclusions:

- "Maori were under-represented in the separate jury pools..."
- "Nationally, 350 Maori were expected to be in the pool of potential jurors .... The actual number was 300, 86 percent of the expected number"

A rather different conclusion to that gained when only the national percentage of Maori in the jury pool was considered.

Lessons

I'd like to draw two conclusions from this example.

• Consider the level of analysis

When we approach data, especially cross-classified data like this, we need to take care about the level at which we analyse. There may be a variable or factor lurking beneath a combined table that hasn't been considered. In this case, the factor was the larger size of jury pools in areas with a higher Maori proportion in the population. It may be desirable to check important results at lower levels of aggregation.

Aggregation can often lead to losing the plot. The situation is not often as clear as with the example we've just seen. But it can be important.

• Check out unexpected results

If you find an interesting, unusual, or unexpected result in analysis, then dig or delve a bit. More than likely there'll be some error or accident contributing to it. But sometimes you'll find something important in its own right.

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Appendix 1:

Why is it called Simpson's Paradox?

Edward Hugh Simpson B.Sc. was elected a fellow of the Royal Statistical Society in 1945-6. He published his paper on the paradox in 1951. Good and Mittal (1987) discuss the origins of the recognition of the paradox involving some of the founders of modern statistics. They provide historical references and document the causal chain Pearson-Yule-Kendall-Simpson. Blyth (1972) called it "Simpson's paradox" in accordance with Stigler's law that the naming of things like this is always wrong.